Risk Loads

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Insurance Risk Triangle

- We jointly model all risks by capturing them in a single data triangle by policy year.
- Reserving risk: allocate accident year losses, by age to policy year. Develop them to ultimate using accident year age to ultimate development factors.
- Pricing risk: (1-expense ratio)* unearned premium reserves = Trimmed UEPR. This is then earned over time. For example, if the premium is earned in 12 months, then all premium income for that policy year will be earned in 24 months.
- The interest rate risk is finally incorporated into this data triangle. To do so, we first recognize that interest rates impact diagonal values only.

Interest Rate Risk: Insurance Risk Triangle

- We also need investment income offset underlying rates by policy year and the actual earned portfolio interest rate. The latter is simply obtained from the accounting department and is a ratio of ending versus initial portfolio values underlying unearned premium/loss reserves.
- If the actual interest rate is higher than expected, we decrease the diagonal to reflect the added investment income. The converse is also true. This way, all the diagonals are restated. The last diagonal is left unchanged as no data for actual interest rate is available as yet.

Illustration: Insurance Risk Triangle

FIGURE 1: INSURANCE RISK TRIANGLE: [M x M+1]

	1	2	3	…	$M-i+1$	$M-i+2$	M-1	M	M+1
1	$U_{1,1}$	$U_{1,2}$	$U_{1,3}$	…	$U_{1,M-i+1}$	$U_{1,M-i+2}$	$U_{1,M-1}$	$U_{1,M}$	$U_{1,M+1}$
2	$U_{2,1}$	$U_{2,2}$	$U_{2,3}$	…	$U_{2,M-i+1}$	$U_{2,M-i+2}$	$U_{2,M-1}$	$U_{2,M}$	
3	$U_{3,1}$	$U_{3,2}$	$U_{3,3}$	…	$U_{3,M-i+1}$	$U_{3,M-i+2}$	$U_{3,M-1}$		
…	…	…	…	…	…	…			
i	$U_{i,1}$	$U_{i,2}$	$U_{i,3}$		$U_{i,M-i+1}$	$U_{i,M-i+2}$			
$i+1$	$U_{i+1,1}$	$U_{i+1,2}$	$U_{i+1,3}$		$U_{i+1,M-i+1}$				
…									
M	$U_{M,1}$	$U_{M,2}$							

Illustration: Insurance Risk Triangle

FIGURE 2: ERROR TRIANGLE: [M x M]

	1	2	…	$M-i$	$M-i+1$	…	M-1	M	
1	$E_{1,1}$	$E_{1,2}$	…	$E_{1,M-i}$	$E_{1,M-i+1}$	…	$E_{1,M-1}$	$E_{1,M}$	
2	$E_{2,1}$	$E_{2,2}$	…	$E_{2,M-i}$	$E_{2,M-i+1}$	…	$E_{2,M-1}$		
…	…	…	…	…	…				
i	$E_{i,1}$	$E_{i,2}$	…	$E_{i,M-i}$	$E_{i,M-i+1}$				
$i+1$	$E_{i+1,1}$	$E_{i+1,2}$	…	$E_{i+1,M-i}$					
…	…	…	…						
M	$E_{M,1}$								

Distribution of Ultimate Loss Ratios

- $U_{i,j} =$ Ultimate Loss Ratio for policy year I, and development year j
- $E_{i,j} =$ Error for policy year I and development interval (j-1, j)
- Assumption:
- Data – complete triangle of Errors and Net Insurance

Distribution of Ultimate Loss Ratio Cont.

- $e_{ij} = \ln\left(\frac{U_{ij}}{U_{i,j-1}}\right) \sim N(\mu_{ij}, \sigma_{ij}^2)$
- $e_i = \sum_{j=0}^{J} e_{ij} = \ln\left(\frac{U_i}{u_i}\right) \sim N(\mu_i, \sigma_i^2)$
- $\ln(U_i) \sim N\left(\mu_i + \ln(u_i), \sigma_i^2\right)$
- If $\mu_i = -\sigma_i^2$ implies
- $\ln(U_i) \sim N(\ln(u_i) - \frac{\sigma_i^2}{2}, \sigma_i^2)$

Risk Load Definition

- Risk Load = $R_i = 1 \ - (U_i + e_i)$
- $\ln(U_i) = \ln(1 \ - R_i - e_i)$
- Using Taylor's series expansion,
- $\ln(U_i) = -(R_i + e_i)$ or $R_i = -(\ln u_i + e_i)$

Risk Load Distribution

- $R_i \sim N(-\ln u_i \ - e_i + \ \sigma_i^2, \ \sigma_i^2)$

- $E(R_i) = \ \sigma_i^2 - \ln(u_i) - e_i)$

- $Var(R_i) = \ \sigma_i^2$

Probability of Shortfall

- $P(\,U_i > E(U_i) + E(R_i)\big) = P(\,U_i > 1\; - e_i)$
- $=1\; - \;\Phi\left(\frac{\ln(1-e_i)-\ln(u_i)+\frac{\sigma_i^2}{2})}{\sigma_i}\right)$

Risk Load Capital

- Risk Load Capital = c_i = VaR Loss Ratio - Unearned premium reserve
- VaR Permissible Loss Ratio = $\exp(\ln(u_i) - \frac{\sigma_i^2}{2} + Z_\alpha \sigma_i)$
- $c_i = u_i \exp\left(-\frac{\sigma_i^2}{2} + Z_\alpha \sigma_i\right) - 1$
- Similarly, firm wide Risk Load Capital
- $c = u \exp\left(-\frac{\sigma_i^2}{2} + Z_\alpha \sigma\right) - 1$

Firm-Wide Expense Ratio and Risk Load

- Firm-Wide Expense Ratio: $e = \sum w_i e_i$
- Firm-Wide Risk Load: $R = \sum w_i R_i$
- $u = 1 - E(R) - e$

Return on Risk Load Capital

- Retrun of Risk Load Capital = Y_i= (Risk Load)/(Risk Load Capital)

- Per Line Return: $Y_i = \frac{R_i}{c_i}$

- For Whole Firm: $Y = \frac{R}{c}$